SlideRule: A Domain-Specific Language for Rewrite Rule Inference Using Equality Saturation

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PLSE

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SlideRule:
A Domain-Specific Language for Rewrite Rule Inference Using Equality Saturation

1. Equality Saturation
2. Rewrite Rule Inference
3. The SlideRule DSL

$(a * 2) / 2$ $\quad a$

$(x * y) / z = x * (y / z)$

$x * 2 = x << 1$

$x / x = 1$

$x * y = y * x$

$x * 1 = x$

$x = x * 1$

$(a * 2) / 2$

$a * (2 / 2)$

$a * 1$

a

$(a * 2) / 2$ $\quad$ a

$(x * y) / z = x * (y / z)$

$x * 2 = x \ll 1$

$x / x = 1$

$x * y = y * x$

$x * 1 = x$

$x = x * 1$

$(a * 2) / 2$

$((a * 1) * 2 / 2$

$((a * 1 * 1) * 2) / 2$

$(a * 2) / 2$

$(2 * a) / 2$

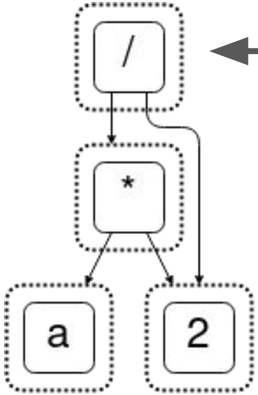
$(a * 2) / 2$

$(a * 2) / 2$

$(a \ll 1) / 2$

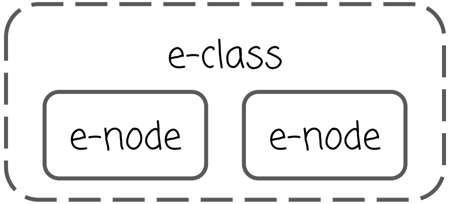
With Equality Saturation,
All of them, All the time

e-graphs

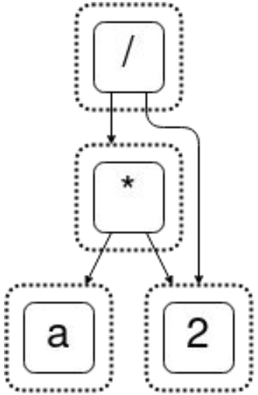


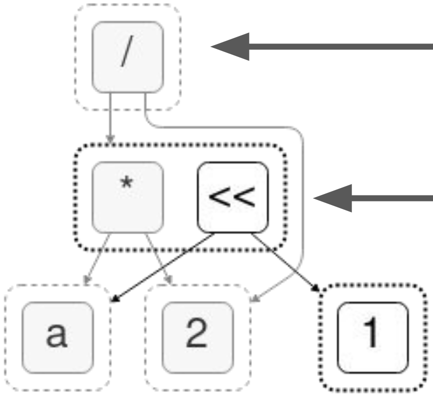
this e-class <u>represents</u>

$$(a * 2) / 2$$



growing an e-graph



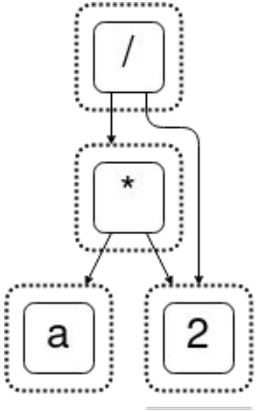


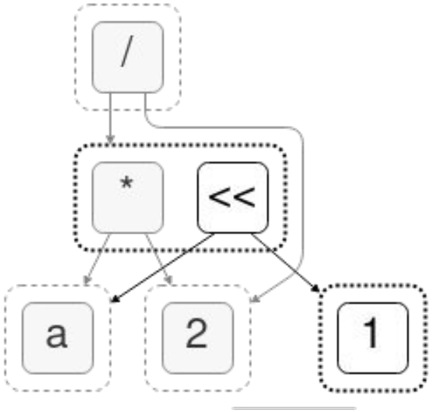
this e-class represents
(a * 2) / 2 **and** (a << 1) / 2

this e-class represents
(a * 2) **and** (a << 1)

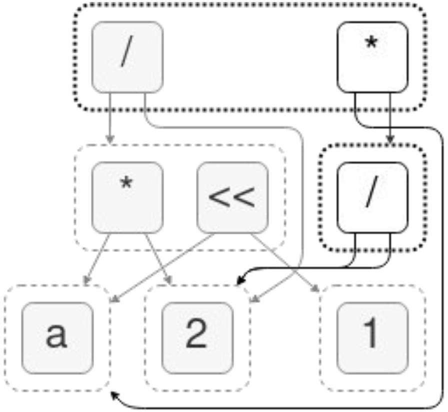
$x * 2 \rightarrow x \ll 1$

growing an e-graph





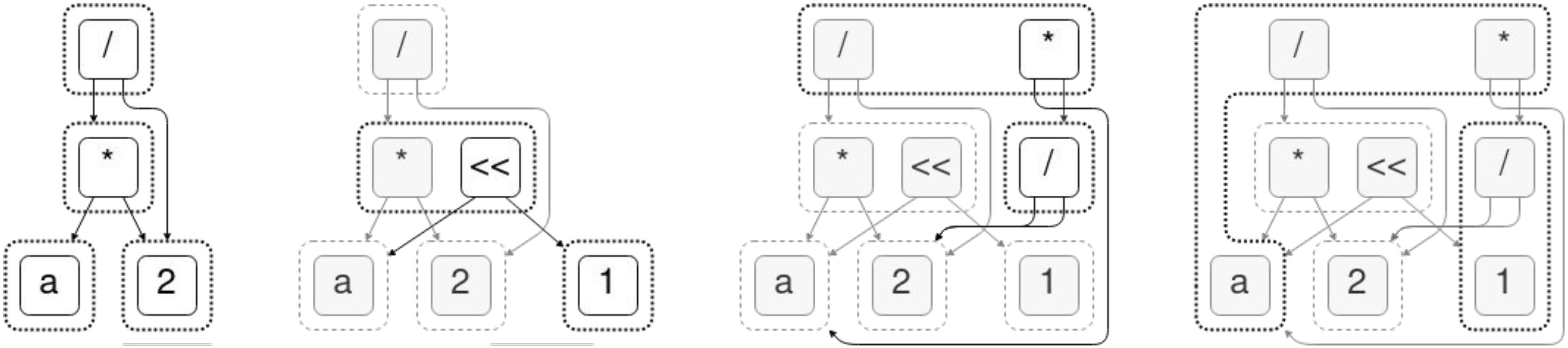
x * 2 → x << 1



(x * y) / z → x * (y / z)

growing an e-graph

a, a * 1,
a * 1 * 1, ...



x * 2 → x << 1

(x * y) / z → x * (y / z)

x / x → 1
x * 1 → x

Using e-graphs requires **high-quality** sets of rewrite rules, which are usually **written by hand** by **domain experts**, making **maintenance difficult**

1. Equality Saturation

2. Rewrite Rule Inference

3. The SlideRule DSL

Rule Inference

1. Term Enumeration
2. Candidate Generation
3. Rule Selection

Term Enumeration

EXPR :=

| var

| number

| (+ EXPR EXPR)

| (* EXPR EXPR)

a, b, 0, 1, (+ a a), (+ a b), (+ a 0), (+ a 1), (+ b a), (+ b b), (+ b 0), (+ b 1), (+ 0 a), (+ 0 b), (+ 0 0), (+ 0 1), (+ 1 a), (+ 1 b), (+ 1 0), (+ 1 1), (* a a), (* a b), (* a 0), (* a 1), (* b a), (* b b), (* b 0), (* b 1), (* 0 a), (* 0 b), (* 0 0), (* 0 1), (* 1 a), (* 1 b), (* 1 0), (* 1 1), (+ a (+ a a)), (+ a (+ a b)), (+ a (+ a 0)), (+ a (+ a 1)), (+ a (+ b a)), (+ a (+ b b)), (+ a (+ b 0)), (+ a (+ b 1)), (+ a (+ 0 a)), (+ a (+ 0 b)), (+ a (+ 0 0)), (+ a (+ 0 1)), (+ a (+ 1 a)), (+ a (+ 1 b)), (+ a (+ 1 0)), (+ a (+ 1 1)), (+ a (* a a)), (+ a (* a b)), (+ a (* a 0)), (+ a (* a 1)), (+ a (* b a)), (+ a (* b b)), (+ a (* b 0)), . . .

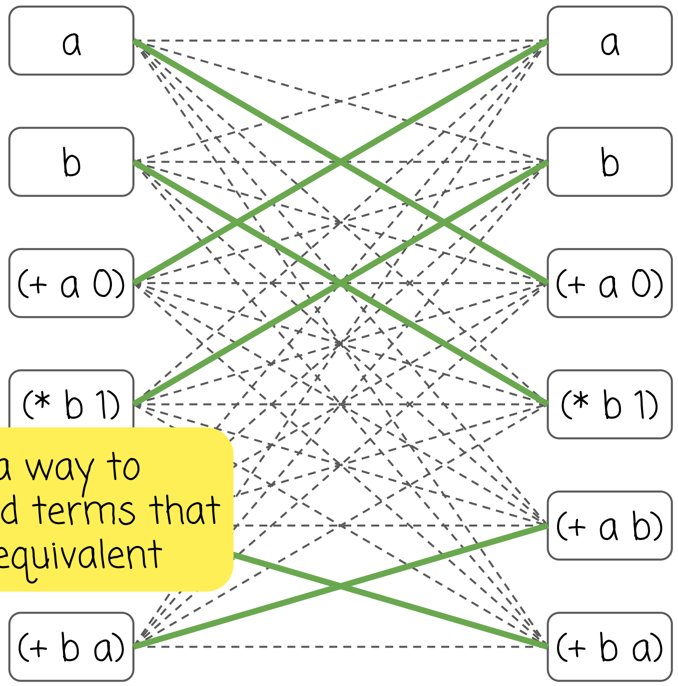
Candidate Generation

Naively, any pair of terms is a potential rewrite rule

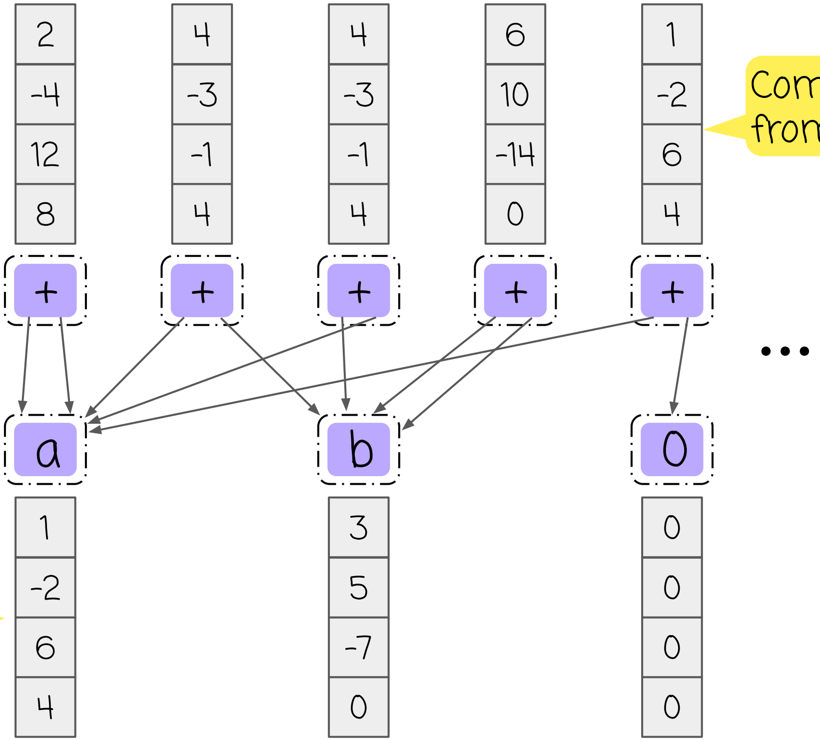
Even for only 6 terms, that leads to 36 potential rules

In this case only 6 of them are actually sound rewrites

We need a way to quickly find terms that are likely equivalent



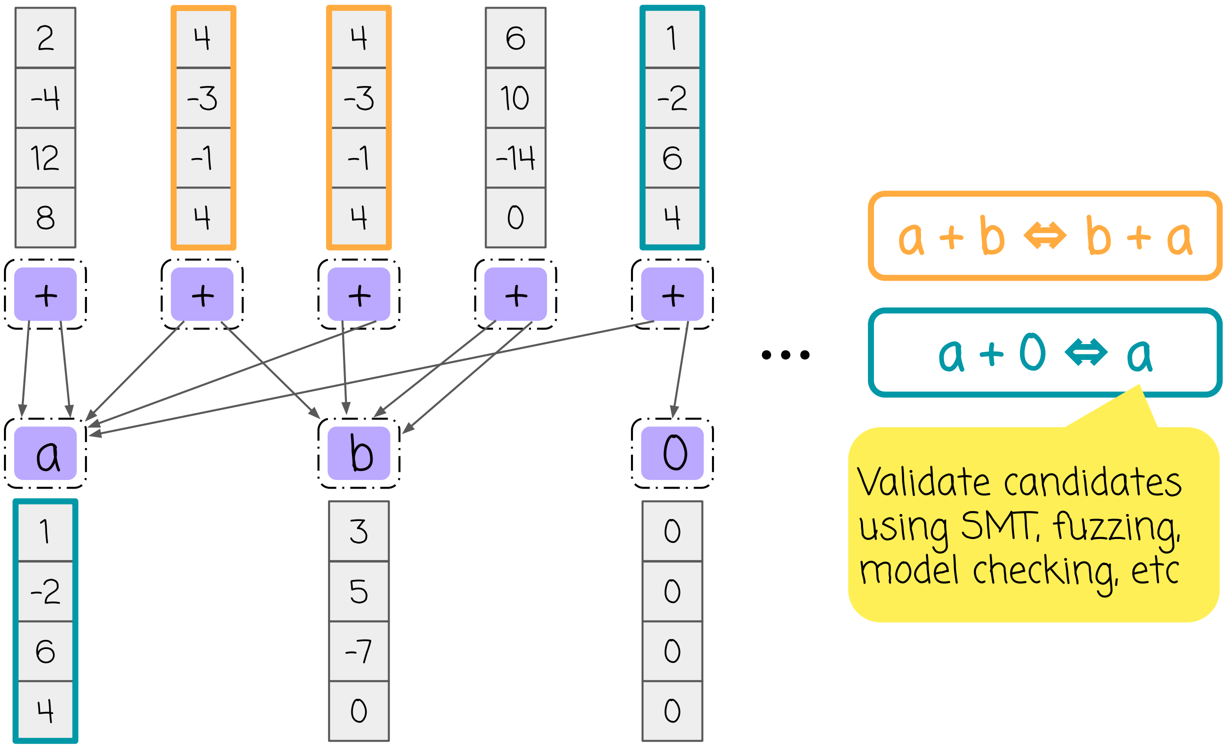
Candidate Generation



Compute cvecs from children

seed variables with concrete values from the domain

Candidate Generation



Rule Selection

$x + y \Leftrightarrow y + x$

$x + 0 \Leftrightarrow 0 + x$

$y + 0 \Leftrightarrow 0 + y$

$x * y \Leftrightarrow y * x$

$x * 1 \Leftrightarrow 1 * x$

$y * 1 \Leftrightarrow 1 * y$

$x + y \Leftrightarrow y + x$

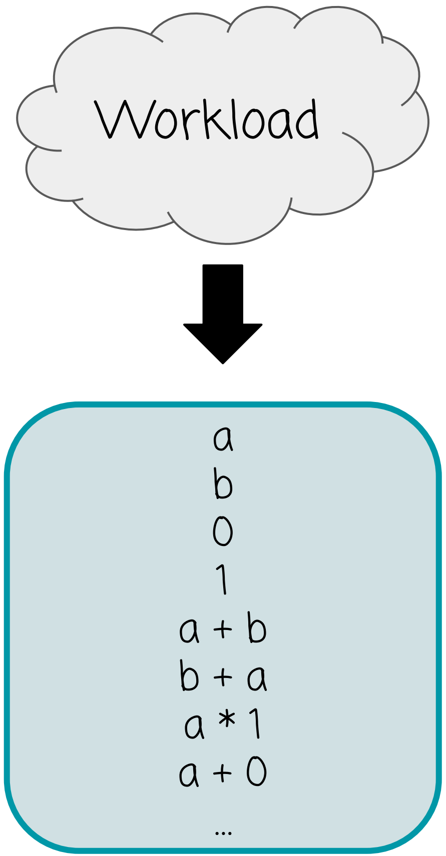
$x * y \Leftrightarrow y * x$

1. Equality Saturation

2. Rewrite Rule Inference

3. The SlideRule DSL

Workloads represent a set of terms



Plug

```
Plug takes two workloads, W1 and W2, and a
string, s

For each term in W1:
    For each occurrence of s in W1:
        For each term, t2, in W2:
            Make a term with t2 substituted for s
```

Plug

```
W1 = { X, foo(X), bar(X, X) }

W2 = { 1, 2, 3 }

Plug(W1, "X", W2)
```

```
1
2
3
```

```
foo(1)
foo(2)
foo(3)
```

```
bar(1, 1)
bar(1, 2)
bar(1, 3)
bar(2, 1)
bar(2, 2)
bar(2, 3)
bar(3, 1)
bar(3, 2)
bar(3, 3)
```

Workloads

```
G = { EXPR (~ EXPR) (+ EXPR EXPR) (* EXPR EXPR) }
leaves = { a b 0 1 }
d2 = Plug(G, "EXPR", leaves)
```

All terms up to depth 2

a	(+ b b)	(* a 0)
b	(+ b 0)	(* a 1)
0	(+ b 1)	(* b a)
1	(+ 0 a)	(* b b)
(~ a)	(+ 0 b)	(* b 0)
(~ b)	(+ 0 0)	(* b 1)
(~ 0)	(+ 0 1)	(* 0 a)
(~ 1)	(+ 1 a)	(* 0 b)
(+ a a)	(+ 1 b)	(* 0 0)
(+ a b)	(+ 1 0)	(* 0 1)
(+ a 0)	(+ 1 1)	(* 1 a)
(+ a 1)	(* a a)	(* 1 b)
(+ b a)	(* a b)	(* 1 0)
		(* 1 1)

Workloads

```
G = { EXPR (~ EXPR) (+ EXPR EXPR) (* EXPR EXPR) }
leaves = { a b 0 1 }
d2 = Plug(G, "EXPR", leaves)
```

`d3 = Plug(G, "EXPR", d2)`

All terms up to depth 3

Everything from d2, and:

```
(~ (~ a))
(~ (~ b))
...
(+ a (+ a a))
(+ a (+ a b))
(+ a (+ 1 1))
...
(+ (* a b) (* a a))
(+ (* a b) (* a b))
...
(* (* 1 1) (* 1 1))
```

Guided Search

```
base = { (OP VAL) }

ops = Plug(base, "OP", { sin cos tan })

all = Plug(ops, "VAL", { (/ PI 2) PI (* 2 PI) })

sound = Filter(all, !Contains("(tan (/ PI 2))"))
```

```
(sin (/ PI 2))   (cos (/ PI 2))   (tan PI)
(sin PI)         (cos PI)         (tan (* PI 2))
(sin (* PI 2))   (cos (* PI 2))
```

Filter out unsound terms

Guided Search

```
prods = Plug({ (* VAL VAL) }, "VAL", { a b 0 1 })

diff_of_prdcts =

   Plug({ (- VAL VAL) }, "VAL", prods)
```

Describe subsets of the term space

```
(* a a)  (* b a)
(* a b)  (* b b)
(* a 0)  (* b 0)
(* a 1)  (* b 1)
     ...
```

```
(- (* a a) (* a a))
(- (* a a) (* a b))
(- (* a a) (* a 0))
(- (* a a) (* a 1))
(- (* a b) (* a a))
        ...
```

Minimize

$x + y \Leftrightarrow y + x$

x

y

$x * y \Leftrightarrow y * x$

$x * 1 \Leftrightarrow 1 * x$

$y * 1 \Leftrightarrow 1 * y$

Rank sound candidates based on generality and select best one

$\boxed{x + y \Leftrightarrow y + x}$

$x * y \Leftrightarrow y * x$

$x + 0 \Leftrightarrow 0 + x$

$y + 0 \Leftrightarrow 0 + y$

$x * 1 \Leftrightarrow 1 * x$

$y * 1 \Leftrightarrow 1 * y$

Minimize

$\boxed{x + y \Leftrightarrow y + x}$

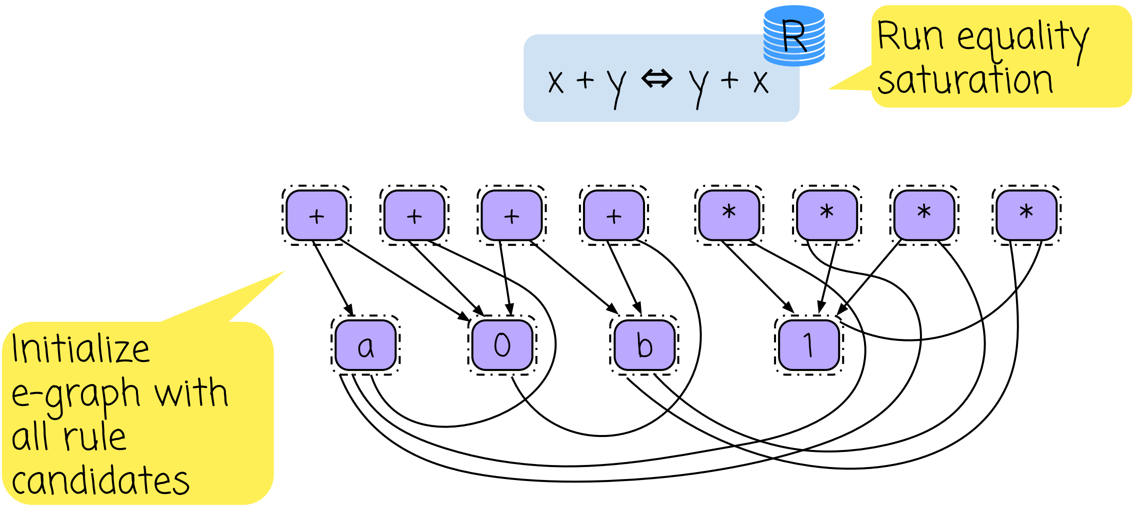
$x * y \Leftrightarrow y * x$

$x + 0 \Leftrightarrow 0 + x$

$y + 0 \Leftrightarrow 0 + y$

$x * 1 \Leftrightarrow 1 * x$

$y * 1 \Leftrightarrow 1 * y$



Minimize

x + y ⇔ y + x

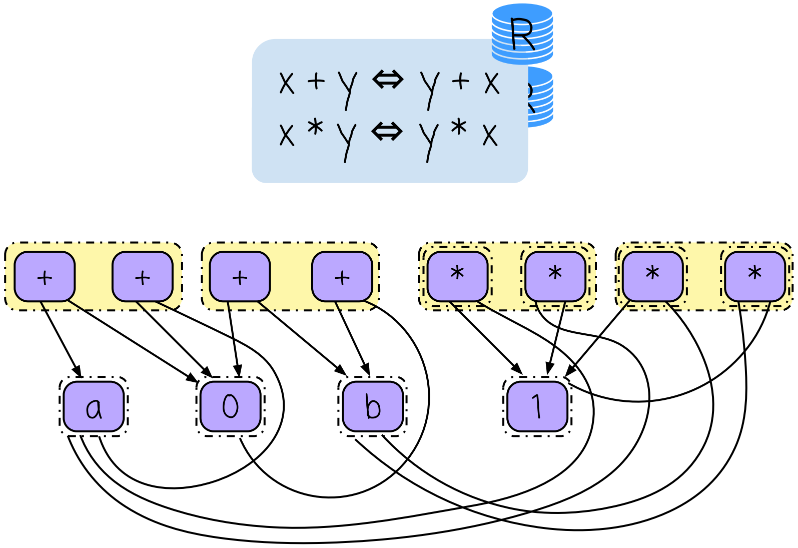
x * y ⇔ y * x

~~x + 0 ⇔ 0 + x~~

~~y + 0 ⇔ 0 + y~~

~~x * 1 ⇔ 1 * x~~

~~y * 1 ⇔ 1 * y~~



Rulesets

```
wkld = ...
```

Start with any Workload constructed using the operators shown previously

Rulesets

```
wkld = ...

egraph = wkld.to_egraph()
```

Initialize an e-graph that represents all the terms in the workload

Rulesets

```
wkld = ...

egraph = wkld.to_egraph()

candidates = egraph.find_candidates()
```

Find candidates using cvec matching

Rulesets

```
wkld = ...
egraph = wkld.to_egraph()
candidates = egraph.find_candidates()
(sound, unsound) =
   candidates.partition(|r| r.is_sound())
```

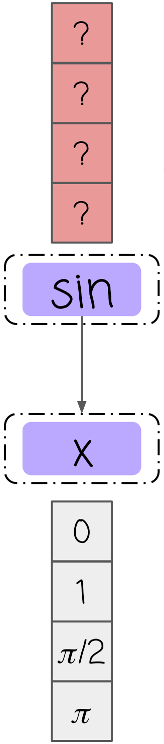
Find sound candidates using the domain-provided rule validator

Rulesets

```
wkld = ...

egraph = wkld.to_egraph()

candidates = egraph.find_candidates()

(sound, unsound) =
   candidates.partition(|r| r.is_sound())

rules = sound.minimize()
```

Minimize the sound candidates to select the best rules

Fast-Forwarding



Cvec matching is not possible in domains where equality is not decidable

Fast-Forwarding

- Run equality saturation in three phases, with different rules in each phase
- Strategically use copies of the e-graph to prevent adding too many new e-nodes and e-classes in each phase
- Learn rule candidates from merged e-classes

Think of these rules as "shortcuts"

Fast-Forwarding

Rules for rational arithmetic

a + b ⇔ b + a
a + (b + c) ⇔ (a + b) + c
a * 0 ⇔ 0
a * 1 ⇔ a
a * (b + c) ⇔ a * b + a * c
…

Rules expressing trig operators in terms of arithmetic

sin x ⇒ (cis x - cis -x) / 2i
cos x ⇒ (cis x + cis -x) / 2
tan x ⇒ sin x / cos x
…

Workload with trig terms

0	tan x	cos 0
1	sin PI	tan 0
PI	cos PI	sin PI/2
sin x	tan PI	cos PI/2
cos x	sin 0	tan PI/2

…

Fast-Forwarding

cos(PI/2 - x) <=> sin x

(1 - (cos(2x))) / 2 <=> $\sin^2 x$

(1 + (cos(2x))) / 2 <=> $\cos^2 x$

sin x * sin y <=> (cos(y - x) - cos(x + y)) / 2

cos x * cos y <=> (cos(x + y) + cos(y - x)) / 2

(cos x * cos y) - (sin x * sin y) <=> cos(x + y)

(sin x * cos y) + (sin y * cos x) <=> sin(x + y)

We can learn all of these rules without evaluating a single trig expression https://github.com/uwplse/ruler

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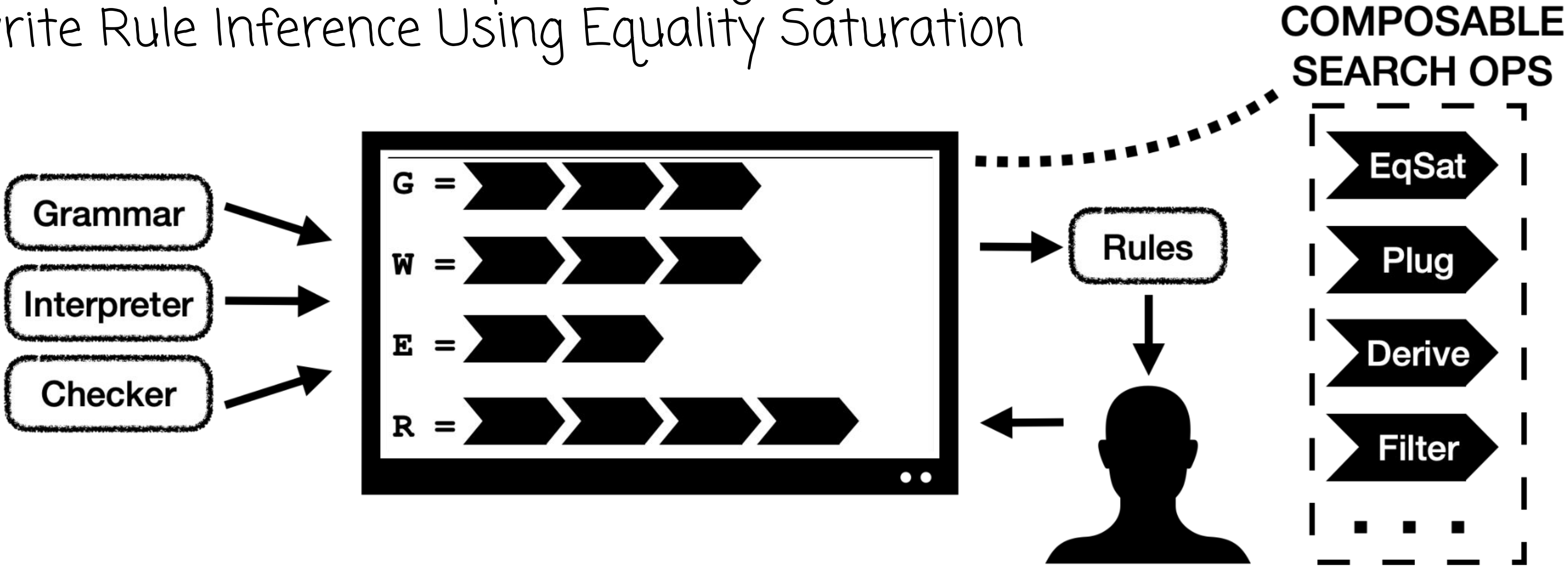









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